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On-Mass-Shell Renormalization of Fermion Mixing Matrices

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Abstract

We consider favourable extensions of the standard model (SM) where the lepton sector contains Majorana neutrinos with vanishing left-handed mass terms, thus allowing for the see-saw mechanism to operate, and propose physical on-mass-shell (OS) renormalization conditions for the lepton mixing matrices that comply with ultraviolet finiteness, gauge-parameter independence, and (pseudo)unitarity. A crucial feature is that the texture zero in the neutrino mass matrix is preserved by renormalization, which is not automatically the case for possible generalizations of existing renormalization prescriptions for the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix in the SM. Our renormalization prescription also applies to the special case of the SM and leads to a physical OS definition of the renormalized CKM matrix.

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1 Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) [1] mixing matrix, which rules the charged-current interactions of the quark mass eigenstates and enables the heavier ones to decay to the lighter ones, is one of the central ingredients of the standard model (SM) of elementary particle physics and, in particular, it is the key to our understanding why certain laws of nature are not invariant under simultaneous charge-conjugation and parity transformations. The CKM matrix is customarily parameterized by three angles and one phase, which constitute four basic parameters of the SM and must be determined by experiment. These parameters represent constants of nature and are listed in the Review of Particle Physics [2]. The CKM matrix elements appear in the bare SM Lagrangian and are thus subject to renormalization. This was realized for the Cabibbo angle in the SM with two fermion generations in a pioneering paper by Marciano and Sirlin [3] and for the CKM matrix of the three-generation SM by Denner and Sack [4] more than a decade ago. So far, all experimental determinations of CKM matrix elements are based on formulas that do not take this into account [2,5].

In quantum electrodynamics, it is very natural and convenient to choose the on-mass-shell (OS) renormalization scheme, which uses the fine-structure constant measured in Thomson scattering and the pole masses of the physical particles as basic parameters. When one attempts to generalize this renormalization scheme to the SM, one also needs to specify a suitable, physically motivated renormalization condition for the CKM matrix. What are the desired properties of the latter? As usual, we split the bare CKM matrix elements V_{ij}^0 , which appear in the original SM Lagrangian, into their renormalized counterparts V_{ij} and the counterterms δV_{ij} as $V_{ij}^0 = V_{ij} + \delta V_{ij}$. Here, i and j label the generations of the up-type and down-type quarks, respectively. To start with, we remark that the parameters V_{ij}^0 are ultraviolet (UV) divergent beyond the tree level and gauge independent. Furthermore, they form a unitary matrix,

$$\sum_k V_{ik}^0 V_{kj}^{0\dagger} = \sum_k V_{ik}^{0\dagger} V_{kj}^0 = \delta_{ij}, \quad (1)$$

which follows from their very definition, $V_{ij}^0 = \sum_k U_{ik}^{0,u} U_{kj}^{0,d\dagger}$, in terms of the unitary matrices $U^{0,u}$ and $U^{0,d}$ that rotate the weak-interaction eigenstates of the bare left-handed up-type and down-type quark fields, respectively, into their mass eigenstates. The UV divergences of V_{ij}^0 are *a priori* unknown.

1. Clearly, δV_{ij} must cancel the UV divergences that, upon coupling and mass renormalization, are left in the loop-corrected amplitude of an arbitrary physical process involving quark mixing. This requirement fixes the UV divergences of δV_{ij} . Different renormalization schemes then differ in the finite parts of δV_{ij} .
2. Apart from being finite, the parameters V_{ij} should also be gauge independent, so that they qualify as proper physical observables that can be extracted from experiment with reason. There is a yet more fundamental reason for this requirement. In fact, for one physical process, namely the decay $W^+ \rightarrow u_i \bar{d}_j$, where u_i and d_j

denote generic up-type and down-type quarks, respectively, it was shown by explicit calculation to one loop in the OS renormalization scheme adopting the R_ξ gauge [6] that the loop-corrected transition (T) matrix element is gauge independent (but UV divergent) if all counterterms are included, except for δV_{ij} [7–10]. Consequently, the loop-corrected result for the partial width of the decay $W^+ \rightarrow u_i \bar{d}_j$ would be gauge dependent if δV_{ij} were. However, this must not be the case.

3. On general grounds, renormalization should be arranged so that the basic structure of the theory is preserved. Since the bare CKM matrix is unitary, the same should, therefore, be true for its renormalized version. Otherwise, four real input parameters would not be sufficient to parameterize the latter, and the familiar notion of unitary triangle would discontinue to be meaningful beyond the tree level. In turn, this would jeopardize the Becchi-Rouet-Stora [11] symmetry of the theory [12]. The unitarity of the renormalized CKM matrix also follows from the request that the commutation relations of the local functional operators related to the Ward-Takahashi [13] identities of the theory formulated with background fields be preserved [8]. At one loop, this leads us to require that

$$\sum_k \left(\delta V_{ik} V_{kj}^\dagger + V_{ik} \delta V_{kj}^\dagger \right) = \sum_k \left(\delta V_{ik}^\dagger V_{kj} + V_{ik}^\dagger \delta V_{kj} \right) = 0 \quad (2)$$

is valid up to higher-order terms.

In summary, a reasonable OS renormalization prescription for the CKM matrix should be physically motivated and satisfy the three requirements enumerated above: UV finiteness, gauge independence, and unitarity. For aesthetical reasons, we wish to add the optional requirement that all pairs (i, j) be treated on a democratic footing.

The OS renormalization prescription for the CKM matrix proposed in Ref. [4] is compact and plausible, complies with the first and third criteria by construction, but — at first sight surprisingly — it fails to satisfy the second criterion because the finite terms of the proposed expressions for δV_{ij} are gauge dependent, as was noticed only recently [7–10]. Obviously, this problem can be circumvented by adopting the modified minimal-subtraction ($\overline{\text{MS}}$) renormalization scheme [14] in dimensional regularization [15], where one only retains the UV divergences of δV_{ij} , proportional to $2/(4-D) + \ln(4\pi) - \gamma_E$. Here, D is the space-time dimensionality, and γ_E is Euler’s constant. In Refs. [7,8], an alternative, OS-like prescription was proposed that avoids this problem at one loop. The characteristic feature of this prescription is that the quark self-energies that enter the definitions of δV_{ij} are not evaluated on their respective mass shells, but at the common subtraction point $q^2 = 0$. In Ref. [9], this prescription was adopted to calculate the partial decay widths of the W boson at one loop in the OS renormalization scheme. Recently, two further prescriptions were introduced [10,16]. The prescription of Ref. [10] is formulated with reference to the case of zero mixing. As will be demonstrated in Section 2, it does not comply with the third criterion. However, we will explain how this drawback can be eliminated. In Ref. [16], the prescription of Ref. [4] was modified by rearranging the

off-diagonal quark wave-function renormalization constants in a manner similar to the pinch technique so that the second criterion is satisfied.

Although the renormalization of the CKM matrix is relevant from the conceptual point of view, its phenomenological significance is damped by the fact that the resulting one-loop effects are at most of order $(\alpha/\pi)m_b^2/M_W^2 \approx 10^{-5}$, where α is the fine-structure constant and m_b and M_W are the masses of the bottom quark and the W boson, respectively [4,9]. This may be understood by observing that, in the approximation of neglecting the masses of the down-type quarks against the W -boson mass, the CKM matrix can be taken to be unity, so that it does not need to be renormalized at all. The situation is possibly very different for lepton mixing in a non-minimal SM with massive Dirac neutrinos or in extensions of the SM involving Majorana neutrinos. We recall that the three neutrino flavours of the minimal SM are strictly massless by construction, due to the absence of right-handed neutrino states.

Recent experiments with solar and atmospheric neutrinos [17] suggest that the known three neutrinos have nonvanishing, yet very small masses and oscillate. In fact, there are experimental indications [17] that mixing is maximal. An attractive theoretical framework for such a scenario is provided by the see-saw mechanism [18], which requires the existence of right-handed neutrino states in addition to the left-handed ones of the minimal SM. A suitably extended SM Lagrangian contains very large right-handed Majorana-neutrino masses, which, together with Dirac-neutrino masses of the order of the charged-lepton or quark masses, form the non-vanishing entries of the see-saw mass matrix. On the other hand, left-handed Majorana-neutrino mass terms are absent, since they would break the gauge invariance of the theory [22]. This leads to the typical texture zero in the see-saw mass matrix. Diagonalization of the latter then naturally gives rise to non-zero, but very small masses for the three known neutrinos, in agreement with experiment, as well as to ultra-heavy neutrinos, which have not yet been discovered. The Lagrangian of this class of Majorana-neutrino theories may be found in Ref. [19]; see also Ref. [20]. Since lepton mixing effects are essential in such extensions of the SM, it is indispensable to renormalize the lepton mixing matrices, not only from the conceptual point of view, but also from the phenomenological one.

In Ref. [20], the OS renormalization prescription of Ref. [4] was extended to general theories with interfamily mixing of Dirac and/or Majorana fermions. By construction, this extended prescription complies with the first and third criteria. However, we found [21] that the failure of the prescription of Ref. [4] to satisfy the second criterion carries over to the one of Ref. [20]. Of course, this problem could be avoided by adopting the $\overline{\text{MS}}$ renormalization scheme [14].

In this paper, we propose a novel OS renormalization prescription for the lepton mixing matrices of Majorana-neutrino theories that is physically motivated and complies with all three criteria [21]. We concentrate on the see-saw scenario described above, which is arguably most appropriate to describe the present experimental situation. A crucial feature of our prescription is that the texture zero in the neutrino mass matrix is preserved by renormalization. We stress that this additional condition is in general not fulfilled for possible generalizations of existing renormalization prescriptions for the CKM matrix.

Our prescription also applies to the special case of the SM and leads to a new OS definition of the renormalized CKM matrix.

This paper is organized as follows. In Section 2, we develop our new OS renormalization prescription for the special case of the CKM matrix in the SM. In Section 3, we generalize it to the Majorana-neutrino theories based on the see-saw mechanism described above. Section 4 contains our conclusions.

2 Renormalization of the CKM matrix in the SM

In this section, we reconsider the partial width of the decay $W^+ \rightarrow u_i \bar{d}_j$ at one loop in the SM and develop a new OS renormalization prescription for the CKM matrix. We adopt the notation from Ref. [9]. The one-loop-corrected T -matrix element has the following structure [9]:¹

$$\mathcal{M}_1^{W u_i d_j} = -\frac{e V_{ij}}{\sqrt{2} s_w} \left\{ \mathcal{M}_1^- \left[1 + \frac{\delta e}{e} - \frac{\delta s_w}{s_w} + \frac{\delta V_{ij}}{V_{ij}} + \frac{1}{2} \delta Z_W \right. \right. \\ \left. \left. + \frac{1}{2 V_{ij}} \sum_k \left(\delta Z_{ik}^{u,L\dagger} V_{kj} + V_{ik} \delta Z_{kj}^{d,L} \right) \right] + \sum_{a=1}^2 \sum_{\sigma=\pm} \mathcal{M}_a^\sigma \delta F_a^\sigma (M_W, m_{u,i}, m_{d,j}) \right\}, \quad (3)$$

where $e = \sqrt{4\pi\alpha}$ is the electron charge magnitude, $s_w = \sin \theta_w$ is the sine of the weak mixing angle, δZ_W is the W -boson wave-function renormalization constant, $\delta Z_{ij}^{u,L}$ and $\delta Z_{ij}^{d,L}$ are the left-handed wave-function renormalization constants for the up-type and down-type quarks, respectively, \mathcal{M}_a^σ are standard matrix elements expressed in terms of the four-momenta, polarization four-vectors, and spinors of the W boson and the u_i and \bar{d}_j quarks, and δF_a^σ are electroweak form factors arising from the proper vertex corrections. The label $\sigma = \pm$ refers to right-handed/left-handed chirality. Notice that the proper vertex corrections only depend linearly on V_{ij} , which is factored out in Eq. (3), because, due to electric-charge conservation, there is just one $W^+ u_i \bar{d}_j$ vertex in each one-loop triangle diagram. We stress that the linear dependence of the vertex corrections on the fermion mixing matrix is a special feature of the SM at one loop, which ceases to be true at higher orders in the SM or even at one loop in general extensions of the SM involving Majorana neutrinos. The renormalization constants δe , δs_w , δZ_W , $\delta Z_{ij}^{u,L}$, and $\delta Z_{ij}^{d,L}$ are all uniquely defined in the electroweak OS renormalization scheme [23]. In the following, we determine δV_{ij} from a physical OS renormalization condition so that all three criteria enumerated in Section 1 are satisfied.

In order to fix δV_{ij} , we proceed in two steps. In the first step, we impose a physical OS renormalization condition to construct an intermediate expression $\delta \tilde{V}_{ij}$ that contains the correct UV divergences and is gauge independent. In other words, we split $V_{ij}^0 = \tilde{V}_{ij} + \delta \tilde{V}_{ij}$ in such a way that \tilde{V}_{ij} satisfies the first and second criteria, but not necessarily the third

¹In Ref. [9], the expression in the second line of Eq. (2.6) should be multiplied by the overall factor $1/V_{ij}$.

one. In the second step, we shift $\delta\tilde{V}_{ij}$ by UV-finite, gauge-independent terms, so that also the third criterion is satisfied.

Our formalism to fix $\delta\tilde{V}_{ij}$ is based on the simple observation that, in the presence of quark mixing, the mapping of up-type and down-type quark mass eigenstates into doublets is completely arbitrary. In the standard nomenclature, the quark mass eigenstates are associated with fermion generations in the order of their masses. However, this is but a convention, albeit a reasonable one. Our proposal is to define $\delta\tilde{V}_{ij}$ by matching Eq. (3) with its counterpart in the theory that emerges from the SM by turning off quark mixing and treating u_i and d_j as isopartners. To be specific, the bare Lagrangian of this modified theory emerges from the one of the SM by taking the bare CKM matrix to be the unit matrix and interchanging the down-type quark fields d_i and d_j if $i \neq j$. This is equivalent to substituting in the bare SM Lagrangian the expression

$$V_{kl}^0 = \delta_{ij}\delta_{kl} + (1 - \delta_{ij})(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} + \epsilon_{ijk}\epsilon_{ijl}), \quad (4)$$

where it is understood that indices that appear in a product more than once are not summed over. Notice that i and j are singled out in this modified theory. Each particular choice of i and j defines a different such theory. Since the bare CKM matrix of Eq. (4) only contains the entries zero and one, it does not need to be renormalized. Thus, the OS renormalization of the modified theory is uniquely fixed by the well-established procedure [23]. In particular, the renormalized electron charge magnitude and particle masses are identified with the respective constants of nature. In turn, this implies that they coincide with those of the SM. On the other hand, the renormalization constants of the parameters and fields of the modified theory will in general differ from their counterparts in the SM because the Feynman rules for the $W^+u_k\bar{d}_l$ and $W^-\bar{u}_kd_l$ vertices are different. As for the parameters, this must be compensated by appropriate shifts in the bare quantities. Henceforth, we denote the renormalization constants and bare parameters of the modified theory by a caret. In particular, we have $e = e^0 - \delta e = \hat{e}^0 - \delta\hat{e}$ and $s_w = s_w^0 - \delta s_w = \hat{s}_w^0 - \delta\hat{s}_w$.² In the reference theory, Eq. (3) is thus replaced by

$$\begin{aligned} \hat{\mathcal{M}}_1^{Wu_id_j} = & -\frac{e}{\sqrt{2}s_w} \left\{ \mathcal{M}_1^- \left[1 + \frac{\delta\hat{e}}{e} - \frac{\delta\hat{s}_w}{s_w} + \frac{1}{2} \left(\delta\hat{Z}_W + \delta\hat{Z}_{ii}^{u,L} + \delta\hat{Z}_{jj}^{d,L} \right) \right] \right. \\ & \left. + \sum_{a=1}^2 \sum_{\sigma=\pm} \mathcal{M}_a^\sigma \delta F_a^\sigma(M_W, m_{u,i}, m_{d,j}) \right\}, \end{aligned} \quad (5)$$

where we have exploited the fact that $\delta\hat{Z}_{ii}^{u,L}$ is real. We stress that all quantities carrying a caret implicitly depend on the specific choice of i and j , via $m_{u,i}$ and $m_{d,j}$. This could be indicated by endowing them with the label (i, j) , which we omit for the time being. This is important to remember when these quantities are to be summed over.

We then impose the physical OS renormalization condition

$$\tilde{\mathcal{M}}_1^{Wu_id_j} = \tilde{V}_{ij} \hat{\mathcal{M}}_1^{Wu_id_j}, \quad (6)$$

²At one loop, we have $\delta\hat{e} = \delta e$ and $\hat{e}^0 = e^0$, since the CKM matrix does not yet enter the electric-charge renormalization.

where $\tilde{\mathcal{M}}_1^{Wu_id_j}$ is obtained from $\mathcal{M}_1^{Wu_id_j}$ of Eq. (3) by replacing V_{ij} and δV_{ij} with \tilde{V}_{ij} and $\delta\tilde{V}_{ij}$, respectively. Notice that, at one loop, the renormalization constants in Eq. (3) are not yet affected by these substitutions. The salient point is that $\hat{\mathcal{M}}_1^{Wu_id_j}$ is UV finite and gauge independent, since it represents the OS-renormalized T -matrix element of a physical process. If we require \tilde{V}_{ij} to be also UV finite and gauge independent, as we do, then Eq. (6) provides an implicit definition of $\delta\tilde{V}_{ij}$ with the desired properties. In the present case, it is particularly simple to solve Eq. (6) for $\delta\tilde{V}_{ij}$, since the form factors δF_a^σ do not depend on CKM matrix elements. We have

$$\frac{\delta\tilde{V}_{ij}}{\tilde{V}_{ij}} = C + \frac{1}{2} \left(\delta\hat{Z}_{ii}^{u,L} + \delta\hat{Z}_{jj}^{d,L} \right) - \frac{1}{2\tilde{V}_{ij}} \sum_k \left(\delta Z_{ik}^{u,L\dagger} \tilde{V}_{kj} + \tilde{V}_{ik} \delta Z_{kj}^{d,L} \right), \quad (7)$$

where

$$C = \frac{\delta\hat{e} - \delta e}{e} - \frac{\delta\hat{s}_w - \delta s_w}{s_w} + \frac{1}{2} \left(\delta\hat{Z}_W - \delta Z_W \right). \quad (8)$$

In fact, Eq. (7) contains the correct UV divergences and is gauge independent, as we checked by explicit calculation. An appealing feature of Eq. (7) is that it only depends on self-energies, while the vertex corrections, which are specific for the considered process, have cancelled. In this formulation of OS renormalization scheme, all one-loop renormalization constants are thus expressed in terms of self-energies. We note in passing that we recover Eq. (25) of Ref. [10] by putting $C = 0$ in Eq. (7). In other words, the shifts in the renormalization constants of the parameters and the W -boson wave function are not taken into account in Ref. [10]. However, at one loop, we have $\delta\hat{e} - \delta e = 0$, while $\delta\hat{s}_w - \delta s_w$ and $\delta\hat{Z}_W - \delta Z_W$ are by themselves UV finite and gauge independent, so that the same is true for C . At one loop, it is, therefore, legitimate to put $C = 0$ in Eq. (7). The price to pay is that the physical OS renormalization condition of Eq. (6) must be surrendered. Although, at first sight, Eq. (7) looks rather complicated, it is easy to implement in practice, the result being just the right-hand side of Eq. (6).

Unfortunately, Eq. (7) and its simplified version of Ref. [10] violate the unitarity condition of Eq. (2), which we checked by explicit calculation. This may also be seen by writing Eq. (7) in the form [4]

$$\delta\tilde{V}_{ij} = \sum_k \left(U_{ik}^\dagger \tilde{V}_{kj} + \tilde{V}_{ik} D_{kj} \right), \quad (9)$$

where

$$\begin{aligned} U_{ik}^\dagger &= \frac{1}{2} \left[\left(C + \delta\hat{Z}_{ii}^{u,L} \right) \delta_{ik} - \delta Z_{ik}^{u,L\dagger} \right], \\ D_{kj} &= \frac{1}{2} \left[\left(C + \delta\hat{Z}_{jj}^{d,L} \right) \delta_{kj} - \delta Z_{kj}^{d,L} \right]. \end{aligned} \quad (10)$$

The unitarity of \tilde{V}_{ij} would be guaranteed if U_{ik}^\dagger and D_{kj} were antihermitian matrices. However, they are not, which already follows from the observation that their diagonal elements are not purely imaginary, even if the real quantity C is nullified.

We obtain our final expression for δV_{ij} by shifting $\delta \tilde{V}_{ij}$ as

$$\delta V_{ij} = \frac{1}{2} \left(\delta \tilde{V}_{ij} - \sum_{k,l} V_{ik} \delta \tilde{V}_{kl}^\dagger V_{lj} \right), \quad (11)$$

which has the same UV divergences as $\delta \tilde{V}_{ij}$, is gauge independent, and exactly satisfies Eq. (2). Notice that, if $\delta \tilde{V}_{ij}$ could be represented in the form of Eq. (9) with antihermitian matrices U^\dagger and D , the right-hand side of Eq. (11) would be equal to $\delta \tilde{V}_{ij}$. Since such a representation is, in fact, possible for the UV divergences of $\delta \tilde{V}_{ij}$, this explains why δV_{ij} has the same UV divergences as $\delta \tilde{V}_{ij}$. We remark that Eq. (11) represents the infinitesimal form of the polar decomposition $\tilde{V} = V |\tilde{V}|$, where $|\tilde{V}| = (\tilde{V}^\dagger \tilde{V})^{1/2}$ [24]. In fact, inserting $\tilde{V} = V^0 - \delta \tilde{V}$ and $V = V^0 - \delta V$ into $V = \tilde{V} (\tilde{V}^\dagger \tilde{V})^{-1/2}$ and neglecting terms beyond one loop, we recover Eq. (11).

Inserting Eq. (7) into Eq. (11) and observing that $V_{ij} = \tilde{V}_{ij} = V_{ij}^0$ at the tree level, we obtain

$$\delta V_{ij} = V_{ij} A(i, j) - \sum_{k,l} V_{ik} V_{kl}^\dagger V_{lj} A(l, k) + \frac{1}{4} \sum_k \left[(\delta Z_{ik}^{u,L} - \delta Z_{ik}^{u,L\dagger}) V_{kj} - V_{ik} (\delta Z_{kj}^{d,L} - \delta Z_{kj}^{d,L\dagger}) \right], \quad (12)$$

where

$$A(i, j) = \frac{1}{2} \left(\frac{\delta \hat{e}}{e} - \frac{\delta \hat{s}_w}{s_w} \right) + \frac{1}{4} (\delta \hat{Z}_W + \delta \hat{Z}_{ii}^{u,L} + \delta \hat{Z}_{jj}^{d,L}). \quad (13)$$

Here, we have used the fact that $A(i, j)$ is real. The third term in Eq. (12) agrees with Eq. (3.16) of Ref. [4], which contains the correct UV divergences, but is known to be gauge dependent [7–10]. The sum of the first two terms in Eq. (12) is UV finite and cancels the gauge dependence of the third term. If the up-type (down-type) quarks were mass degenerate, then $A(i, j)$ would be independent of i (j), so that the first two terms in Eq. (12) would cancel. Inserting Eq. (12) into Eq. (3), we would then be left with the hermitian parts of $\delta Z_{ik}^{u,L}$ and $\delta Z_{kj}^{d,L}$, which are regular in the limit where the masses of the up-type or down-type quarks coincide [20]. Consequently, Eq. (3) is regular in this limit, as it should. In the case of exact mass degeneracy, one would, of course, avoid the issue of CKM-matrix renormalization altogether by setting $V_{ij}^0 = \delta_{ij}$ in the bare Lagrangian.

From the discussion below Eq. (8) it follows that, at one loop, we may simplify Eq. (12) by omitting the first three terms on the right-hand side of Eq. (13), at the expense of abandoning Eq. (6). We consider this as *ad hoc*. While the expressions for δV_{ij} proposed in Ref. [8] involve quark self-energies evaluated at $q^2 = 0$, Eq. (12) is constructed from ordinary OS renormalization constants [23] and thus deserves to be referred to as a genuine OS counterterm for the CKM matrix. A similar comment applies to Ref. [16], where the quark self-energies are manipulated by means of a procedure similar to the pinch technique, so that the resulting quark wave-function renormalization constants differ from the conventional OS ones [23].

The decay $W^+ \rightarrow u_i \bar{d}_j$ is kinematically forbidden if $i = 3$. We could then derive Eq. (12) by considering the crossed process, $u_i \rightarrow W^+ d_j$. The advantage of our renormalization procedure is that, owing to its conceptual transparency, it can be extended straightforwardly to extensions of the SM involving Majorana neutrinos. This is the topic of Section 3. Furthermore, we believe that it is likely to carry over to higher orders.

We conclude this section by elaborating an alternative physical OS renormalization condition for the CKM matrix, which was already mentioned in Ref. [4], namely to demand that the loop-corrected T -matrix elements of four selected $W^+ \rightarrow u_i \bar{d}_j$ decays coincide with the respective tree-level expressions. One would reasonably pick those decay channels whose partial widths are most precisely measured. Specifically, one requires for these choices of (i, j) that

$$\mathcal{M}_1^{W u_i d_j} = \mathcal{M}_0^{W u_i d_j}, \quad (14)$$

where $\mathcal{M}_1^{W u_i d_j}$ is given by Eq. (3), $\mathcal{M}_0^{W u_i d_j} = -\left(e V_{ij}/\sqrt{2} s_w\right) \mathcal{M}_1^-$ is the tree-level result written in terms of renormalized parameters, and it is understood that only terms proportional to \mathcal{M}_1^- are retained. The other standard matrix elements only enter at one loop, so that their form factors are UV finite and gauge independent by themselves. From Eq. (3), one gleans that

$$\frac{\delta V_{ij}}{V_{ij}} = -\frac{\delta e}{e} + \frac{\delta s_w}{s_w} - \frac{1}{2} \delta Z_W - \frac{1}{2 V_{ij}} \sum_k \left(\delta Z_{ik}^{u, L\dagger} V_{kj} + V_{ik} \delta Z_{kj}^{d, L} \right) - \delta F_1^- (M_W, m_{u,i}, m_{d,j}). \quad (15)$$

By construction, Eq. (15) contains the correct UV divergences and is gauge independent. The residual five counterterms δV_{ij} must then be fixed so that Eq. (2) is fulfilled. This is conveniently achieved with the aid of the standard parameterization of the CKM matrix, which utilizes three angles, θ_{12} , θ_{23} , and θ_{13} , and one phase, δ_{13} [25]; see also Eq. (11.3) of Ref. [2]. In terms of the bare parameters $(\alpha_1^0, \alpha_2^0, \alpha_3^0, \alpha_4^0) \equiv (\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_{13}^0)$, we can write $V_{ij}^0 = f_{ij}(\boldsymbol{\alpha}^0)$. Substituting $\alpha_k^0 = \alpha_k + \delta\alpha_k$, we can identify $V_{ij} = f_{ij}(\boldsymbol{\alpha})$, so that

$$\delta V_{ij} = \sum_{k=1}^4 \delta\alpha_k \frac{\partial f_{ij}(\boldsymbol{\alpha})}{\partial \alpha_k}, \quad (16)$$

up to higher-order terms. Equating Eqs. (15) and (16) for the four selected pairs (i, j) , we obtain a linear system of equations, which we can solve for $\delta\alpha_k$. The solutions are gauge independent. The counterterms δV_{ij} for the residual pairs (i, j) are then given by Eq. (16). They are gauge independent, too. We checked by explicit calculation that they also contain the correct UV divergences. In conclusion, this alternative OS renormalization condition is physical and satisfies all three criteria.

An alternative formulation that retains all form factors is obtained by taking the absolute square and summing over the polarization of the W^+ boson and the spins of the u_i and \bar{d}_j quarks on both sides of Eq. (14). The resulting expression for $\delta V_{ij}/V_{ij}$ differs from Eq. (15) by the additional term $-\sum_{(a,\sigma)} \left(G_a^\sigma / G_1^- \right) \delta F_a^\sigma (M_W, m_{u,i}, m_{d,j})$, where it is summed over $(a, \sigma) = (1, +), (2, +), (2, -)$ and $G_a^\sigma = \sum_{\text{pol}} \mathcal{M}_1^{-\dagger} \mathcal{M}_a^\sigma$ are real functions of M_W , $m_{u,i}$, and $m_{d,j}$, which may be found in Eq. (2.3) of Ref. [9]. This term is UV

finite and gauge independent. The determination of the residual counterterms δV_{ij} then proceeds as explained above.

Obvious drawbacks of Eq. (14) and its variant discussed in the preceding paragraph are that they destroy the symmetry of Eq. (3) with respect to the quark families [4] and that the resulting expressions for δV_{ij} involve vertex corrections, which are specific for the selected processes, whereas all other renormalization constants of the SM can be expressed in terms of self-energies. While Eq. (6) does not suffer from these drawbacks, it entails the minor complication that one needs to consider a reference theory with zero mixing. Notwithstanding, we find the OS renormalization prescription presented in the first part of this section preferable.

3 Renormalization of the lepton mixing matrices in Majorana-neutrino theories

In this section, we consider a minimal, renormalizable extension of the SM, based on the $SU(2)_I \otimes U(1)_Y$ gauge group, that can naturally accommodate heavy Majorana neutrinos [19,20] and propose physical OS renormalization conditions for its lepton mixing matrices that satisfy all three criteria enumerated in Section 1. To this end, we need to generalize the formalism proposed in Section 2. As a new feature, we encounter the constraint that the texture zero in the see-saw mass matrix should be preserved by the renormalization procedure in order not to increase the number of independent parameters.

To start with, we summarize the basic features of the lepton sector of the Majorana-neutrino theory under consideration, adopting the notation from Refs. [19,20]. For generality, we allow for an arbitrary number N_G of fermion generations. Similarly to the SM, each lepton family contains one weak-isospin (I) doublet $(\nu'_{L,i}, l'_{L,i})$ of left-handed states with weak hypercharge $Y = -1$ and one right-handed charged-lepton state $l'_{R,i}$ with $I = 0$ and $Y = -2$. In addition, there is a total of N_R right-handed neutrinos $\nu'_{R,i}$ with $I = Y = 0$. Here, the primes are to remind us that we are dealing with weak-interaction eigenstates. Deviating from Refs. [19,20], we require that N_R is a multiple of N_G and that there are N_R/N_G right-handed neutrinos in each lepton family, so that all lepton families have the same structure. The quark families are taken to be of the SM type. The bare Lagrangian contains the neutrino mass terms

$$\mathcal{L}_Y^{0,\nu} = -\frac{1}{2} \left(\bar{\nu}_L^{\prime 0}, \bar{\nu}_R^{\prime 0C} \right) M^{0,\nu} \begin{pmatrix} \nu_L^{\prime 0C} \\ \nu_R^{\prime 0} \end{pmatrix} + \text{h.c.}, \quad (17)$$

where $\nu_L^{\prime 0} = (\nu_{L,1}^{\prime 0}, \dots, \nu_{L,N_G}^{\prime 0})^T$, $\nu_R^{\prime 0} = (\nu_{R,1}^{\prime 0}, \dots, \nu_{R,N_R}^{\prime 0})^T$, the superscript C denotes charge conjugation, and $M^{0,\nu}$ is a complex, symmetric mass matrix of the form

$$M^{0,\nu} = \begin{pmatrix} m_L^{\prime 0} & m_D^{\prime 0} \\ m_D^{\prime 0T} & m_M^{\prime 0} \end{pmatrix}. \quad (18)$$

Unless the SM Higgs sector is supplemented by an additional weak-isospin triplet of Higgs fields, gauge invariance enforces $m_L^{\prime 0} = 0$ [22]. In the following, we assume that $m_L^{\prime 0} = 0$.

This allows for the see-saw mechanism [18] to operate. The neutrino mass matrix $M^{0,\nu}$ can always be diagonalized through a unitary transformation $U^{0,\nu}$ as

$$U^{0,\nu T} M^{0,\nu} U^{0,\nu} = \text{diag} \left(m_{n,1}^0, \dots, m_{n,N_G+N_R}^0 \right), \quad (19)$$

where $m_{n,i}^0 \geq 0$ at the tree level. The corresponding mass eigenstates are given by

$$\begin{aligned} \begin{pmatrix} \nu_L^0 \\ \nu_R^{0C} \end{pmatrix}_i &= \sum_{j=1}^{N_G+N_R} U_{ij}^{0,\nu*} n_{L,j}^0, \\ \begin{pmatrix} \nu_L^{0C} \\ \nu_R^0 \end{pmatrix}_i &= \sum_{j=1}^{N_G+N_R} U_{ij}^{0,\nu} n_{R,j}^0. \end{aligned} \quad (20)$$

Here, the first N_G mass eigenstates, $\nu_i \equiv n_i$ ($i = 1, \dots, N_G$), are identified with the ordinary light neutrinos (assuming that $N_G = 3$), and the remaining N_R states, $N_i \equiv n_{N_G+i}$ ($i = 1, \dots, N_R$), represent the new neutral leptons predicted by the theory. The latter are the heavy Majorana neutrinos, which have not yet been discovered. The diagonalization of the charged-lepton mass matrix proceeds as in the quark case discussed in Section 2.

In this Majorana-neutrino theory, mixing effects enter via the interactions of the charged leptons l_i and Majorana neutrinos n_i with the W , Z , and Higgs bosons. The bare Lagrangian of the charged-current interaction involves the $N_G \times (N_G + N_R)$ mixing matrix B^0 , while the ones of the neutral-current and Yukawa interactions involve the $(N_G + N_R) \times (N_G + N_R)$ mixing matrix C^0 . Specifically, we have

$$\begin{aligned} B_{ij}^0 &= \sum_{k=1}^{N_G} V_{ik}^{0,l} U_{kj}^{0,\nu*}, \\ C_{ij}^0 &= \sum_{k=1}^{N_G} U_{ik}^{0,\nu T} U_{kj}^{0,\nu*}, \end{aligned} \quad (21)$$

where $V^{0,l}$ is the unitary $N_G \times N_G$ matrix relating the weak-interaction and mass eigenstates of the bare left-handed charged-lepton fields,

$$l_{L,i}^0 = \sum_{j=1}^{N_G} V_{ij}^{0,l} l_{L,j}^0. \quad (22)$$

Notice that the summations in Eq. (21) stop at $k = N_G$, rather than at $k = N_G + N_R$, thereby projecting the neutrino state vector onto its non-isosinglet components. From Eq. (21), it follows that B^0 is pseudo-unitary, in the sense that it satisfies the relationships

$$\sum_{k=1}^{N_G+N_R} B_{ik}^0 B_{kj}^{0\dagger} = \delta_{ij}, \quad (23)$$

$$\sum_{k=1}^{N_G} B_{ik}^{0\dagger} B_{kj}^0 = C_{ij}^0. \quad (24)$$

From Eq. (21), it also follows that

$$\begin{aligned} C_{ij}^{0\dagger} &= \sum_{k=1}^{N_G+N_R} C_{ik}^0 C_{kj}^0 = C_{ij}^0, \\ \sum_{k=1}^{N_G+N_R} B_{ik}^0 C_{kj}^0 &= B_{ij}^0. \end{aligned} \quad (25)$$

Notice that Eq. (25) can also be derived from Eqs. (23) and (24), reflecting the fact that C^0 is but an auxiliary quantity, which is completely fixed once B^0 is. In the see-saw scenario, with $m_L^0 = 0$, there are additional relationships between B^0 , C^0 , and $m_{n,i}^0$, namely

$$\sum_{k=1}^{N_G+N_R} B_{ik}^0 m_{n,k}^0 B_{kj}^{0T} = 0, \quad (26)$$

$$\sum_{k=1}^{N_G+N_R} B_{ik}^0 m_{n,k}^0 C_{kj}^{0T} = 0, \quad (27)$$

$$\sum_{k=1}^{N_G+N_R} C_{ik}^0 m_{n,k}^0 C_{kj}^{0T} = 0. \quad (28)$$

Notice that Eqs. (27) and (28) follow from Eq. (26) with the aid of Eq. (24). For later use, we remark here that Eqs. (26)–(28) are valid for arbitrary values of $m_{n,i}^0$.

We now turn to the renormalization of the lepton mixing matrices and write $B_{ij}^0 = B_{ij} + \delta B_{ij}$, $C_{ij}^0 = C_{ij} + \delta C_{ij}$, and $m_{n,i}^0 = m_{n,i} + \delta m_{n,i}$. The third criterion implies that the renormalized mixing matrices B and C must satisfy relations analogous to Eqs. (23) and (24). Then, they automatically satisfy relations analogous to Eq. (25). This leads to the following conditions:

$$\sum_{k=1}^{N_G+N_R} \left(\delta B_{ik} B_{kj}^\dagger + B_{ik} \delta B_{kj}^\dagger \right) = 0, \quad (29)$$

$$\sum_{k=1}^{N_G} \left(\delta B_{ik}^\dagger B_{kj} + B_{ik}^\dagger \delta B_{kj} \right) = \delta C_{ij}, \quad (30)$$

up to higher-order terms. Equation (29) is the analogue of Eq. (2) for the CKM matrix, while Eq. (30) tells us that δC_{ij} is fixed once δB_{ij} is. Having found an expression for δB_{ij} that contains the correct UV divergences and satisfies Eq. (29), we are in general faced with the situation that radiative corrections destroy the texture zero in the see-saw mass matrix. It is, therefore, necessary to impose the additional condition that B , C , and $m_{n,i}$ must satisfy relations analogous to Eqs. (26)–(28), in order not to increase the number of independent parameters. As in bare case, it is sufficient to require the analogue of Eq. (26), which is satisfied if

$$\sum_{k=1}^{N_G+N_R} \left(\delta B_{ik} m_{n,k} B_{kj}^T + B_{ik} \delta m_{n,k} B_{kj}^T + B_{ik} m_{n,k} \delta B_{kj}^T \right) = 0, \quad (31)$$

up to higher-order terms. In Ref. [20], this additional condition was not imposed, with the consequence that the second criterion was not fulfilled. This would in general also happen in possible generalizations of the renormalization prescriptions for the CKM matrix recently proposed in Refs. [7,8,10] if this additional condition were not implemented.

In the following, we present a physical OS renormalization prescription for the lepton mixing matrices that satisfies all three criteria and, in particular, guarantees that $m'_L = 0$ if $m_L^{0'} = 0$ is chosen. We start by observing that in the Majorana-neutrino theory under consideration, with $m_L^{0'} = 0$, the parameters B_{ij}^0 are in general functions of $m_{n,k}^0$ and some bare angles θ_l^0 and phases δ_m^0 fixing the remaining degrees of freedom, so that Eqs. (23) and (26) are fulfilled for all values of $m_{n,k}^0$, θ_l^0 , and δ_m^0 . That is, we have some parameterization $B_{ij}^0 = f_{ij}(\mathbf{m}_n^0, \boldsymbol{\alpha}^0)$, where $\boldsymbol{\alpha}^0 = (\boldsymbol{\theta}^0, \boldsymbol{\delta}^0)$. In the most general case, there are $2N_G(N_R - 1)$ independent parameters α_l^0 . On the other hand, the quark mixing matrix of this theory comprises $(N_G - 1)^2$ independent parameters. Henceforth, we confine ourselves to parameterizations of B^0 with the property that $\hat{B}_{ij}^0 = f_{ij}(\mathbf{m}_n^0, \mathbf{0}) \equiv \hat{f}_{ij}(\mathbf{m}_n^0)$ does not mix different lepton families. Although this requirement restricts the choice of parameterizations, we believe that all phenomenologically interesting scenarios can still be described. Notice that \hat{B}^0 does not imply zero mixing, as this would in general violate Eq. (26). This rather means that, depending on the particular choice of parameterization, \hat{B}^0 is as close as possible to the zero-mixing case, while it still satisfies Eq. (26). The only parameters that the bare Lagrangian of the Majorana-neutrino theory with \hat{B}^0 contains in addition to those of the SM are $m_{n,k}^0$. This theory can, therefore, be renormalized according to the well-established OS renormalization scheme [23], without introducing any other additional counterterms than $\delta m_{n,k}$ [20]. Similarly to $(s_w^0)^2 = 1 - (M_W^0)^2 / (M_Z^0)^2$ in the SM, the parameters $\hat{B}_{ij}^0 = \hat{f}_{ij}(\mathbf{m}_n^0)$ are then not treated as basic ones, but rather as convenient abbreviations. Writing $\hat{B}_{ij}^0 = \hat{B}_{ij} + \delta \hat{B}_{ij}$, we can identify $\hat{B}_{ij} = \hat{f}_{ij}(\mathbf{m}_n)$, so that

$$\delta \hat{B}_{ij} = \sum_{k=1}^{N_G + N_R} \delta m_{n,k} \frac{\partial \hat{f}_{ij}(\mathbf{m}_n)}{\partial m_{n,k}}, \quad (32)$$

up to higher-order terms. In this way, \hat{B}_{ij} is automatically UV finite, gauge independent, and pseudo-unitary. Moreover, it satisfies Eq. (26) written with \hat{B}_{ij} and $m_{n,k}$. The counterterms $\delta \hat{B}_{ij}$ cancel the additional UV divergences that arise through the intrafamily lepton mixing induced by \hat{B}^0 .

Next, we return to the theory with B^0 by reintroducing the parameters α_k^0 . Similarly to Section 2, we proceed in two steps to fix δB_{ij} . We first construct intermediate counterterms $\delta \tilde{B}_{ij}$ that contain the correct UV divergences and are gauge independent. To this end, we introduce a physical OS renormalization condition relating the loop-corrected results for the partial width of the decay $W^- \rightarrow l_i^- n_j$ (or $n_j \rightarrow W^+ l_i^-$) calculated in the theories with B^0 and \hat{B}^0 . The resulting expressions for $\delta \tilde{B}_{ij}$ and $\tilde{B}_{ij} = B_{ij}^0 - \delta \tilde{B}_{ij}$ will in general not satisfy the pseudo-unitarity condition of Eq. (29). Then, we obtain our final expression for δB_{ij} by adjusting $\delta \tilde{B}_{ij}$ by UV-finite, gauge-independent terms so that Eq. (29) is fulfilled.

In the general theory with B^0 , the one-loop-corrected T -matrix element of the decay $W^- \rightarrow l_i^- n_j$ has the form

$$\mathcal{M}_1^{Wl_i n_j} = -\frac{eB_{ij}}{\sqrt{2}s_w} \left\{ \mathcal{M}_1^- \left[1 + \frac{\delta e}{e} - \frac{\delta s_w}{s_w} + \frac{\delta B_{ij}}{B_{ij}} + \frac{1}{2}\delta Z_W \right] + \frac{1}{2B_{ij}} \sum_k \left(\delta Z_{ik}^{l,L\dagger} B_{kj} + B_{ik} \delta Z_{kj}^{n,L} \right) \right\} + \sum_{a=1}^2 \sum_{\sigma=\pm} \mathcal{M}_a^\sigma \delta F_a^\sigma, \quad (33)$$

where the form factors δF_a^σ are now functions of $M_W, M_Z, M_H, m_{l,k}, m_{n,l}, B_{mn}$. In the minimal-mixing theory with \hat{B}^0 , the decay $W^- \rightarrow l_i^- n_j$ is only allowed if l_i and n_j belong to the same lepton family. If this is not the case, then we need to redefine this theory by interchanging l_i and l_j . We are entitled to do so because, in the general-mixing theory with B^0 , the assignment of mass eigenstates to fermion generations is, as a matter of principle, arbitrary. The expression for $\hat{\mathcal{M}}_1^{Wl_i n_j}$ in the reference theory thus defined emerges from Eq. (33) by substituting B_{ij} with \hat{B}_{ij} and placing a caret on each renormalization constant. The form factors, which we denote by $\delta \hat{F}_a^\sigma$, are now independent of the masses $m_{l,k}$ and $m_{n,l}$ of those leptons l_k and n_l that do not belong the same family as l_i and n_j . In contrast to $\hat{\mathcal{M}}_1^{Wu_i d_j}$ of Eq. (5), $\hat{\mathcal{M}}_1^{Wl_i n_j}$ contains the overall factor \hat{B}_{ij} , which in general differs from unity. We thus need to generalize the physical OS renormalization condition of Eq. (6) to read

$$\tilde{\mathcal{M}}_1^{Wl_i n_j} = \frac{\tilde{B}_{ij}}{\hat{B}_{ij}} \hat{\mathcal{M}}_1^{Wl_i n_j}, \quad (34)$$

where it is understood that we only retain terms proportional to \mathcal{M}_1^- . The other standard matrix elements only enter at one loop, so that their form factors are UV finite and gauge independent by themselves. Below, we outline an alternative approach that also includes these form factors. We note that $\hat{B}_{ij} \neq 0$ because the reference theory is arranged so that $\hat{B}_{ij}^0 \neq 0$. Solving Eq. (34) for $\delta \tilde{B}_{ij}$, we find

$$\begin{aligned} \frac{\delta \tilde{B}_{ij}}{\tilde{B}_{ij}} = C + \frac{\delta \hat{B}_{ij}}{\hat{B}_{ij}} + \frac{1}{2\hat{B}_{ij}} \sum_k \left(\delta \hat{Z}_{ik}^{l,L\dagger} \hat{B}_{kj} + \hat{B}_{ik} \delta \hat{Z}_{kj}^{n,L} \right) - \frac{1}{2\tilde{B}_{ij}} \sum_k \left(\delta Z_{ik}^{l,L\dagger} \tilde{B}_{kj} + \tilde{B}_{ik} \delta Z_{kj}^{n,L} \right) \\ + \delta \hat{F}_1^- - \delta F_1^-, \end{aligned} \quad (35)$$

where C is defined in Eq. (8). By construction, Eq. (35) contains the correct UV divergences and is gauge independent, as we also checked by explicit calculation for representative choices of B^0 . In contrast to the SM, where all OS renormalization constants can be expressed in terms of self-energies [23], Eq. (35) involves also vertex corrections. Similarly to Eq. (7), at one loop, we can simplify Eq. (35) by discarding C , at the expense of sacrificing Eq. (34).

An alternative formulation that retains all form factors is obtained by taking the absolute square and summing over the polarization of the W^- boson and the spins of the l_i^- and n_j leptons on both sides of Eq. (34). The resulting expression for $\delta \tilde{B}_{ij}/\tilde{B}_{ij}$ differs from Eq. (35) by the additional term $\sum_{(a,\sigma)} \left(G_a^\sigma/G_1^- \right) \left(\delta \hat{F}_a^\sigma - \delta F_a^\sigma \right)$, where it is summed

over $(a, \sigma) = (1, +), (2, +), (2, -)$. Since this term is UV finite and gauge independent, we have the option of discarding it along with C , with the same consequence.

Unfortunately, Eq. (35) and its variants discussed above violate the pseudo-unitarity condition of Eq. (29). Similarly to Eq. (11), this problem can be fixed by the redefinition

$$\delta B_{ij} = \frac{1}{2} \left(\delta \tilde{B}_{ij} - \sum_{k,l} B_{ik} \delta \tilde{B}_{kl}^\dagger B_{lj} \right), \quad (36)$$

which has the same UV divergences as $\delta \tilde{B}_{ij}$, is gauge independent, and exactly satisfies Eq. (29). Inserting Eq. (35) into Eq. (36), we obtain an expression for δB_{ij} that extends Eq. (6.1) of Ref. [20] by UV-finite, gauge-dependent terms, which compensate the gauge dependence of that equation. Our final result for B_{ij} originates from a genuine OS renormalization condition and satisfies all three criteria.

We conclude this section by pointing out that the alternative physical OS renormalization condition for the CKM matrix worked out at the end of Section 2 can straightforwardly be extended to the Majorana-neutrino case. To this end, one chooses a parameterization $B_{ij}^0 = f_{ij}(\mathbf{m}_n^0, \boldsymbol{\alpha}^0)$ that satisfies Eqs. (23) and (26) and selects as many $W^+ \rightarrow l_i^- n_j$ decay channels as there are parameters α_k^0 , namely $2N_G(N_R - 1)$. For these processes, one then defines δB_{ij} by nullifying the loop corrections proportional to \mathcal{M}_1^- in Eq. (33). As in the quark case, one can alternatively equate the absolute squares of the tree-level and loop-corrected T -matrix elements after summation over polarization and spins. The residual expressions for δB_{ij} are then determined according to the procedure outlined at the end of Section 2, except that we now have, up to higher-order terms,

$$\delta B_{ij} = \sum_{k=1}^{N_G+N_R} \delta m_{n,k} \frac{\partial f_{ij}(\mathbf{m}_n, \boldsymbol{\alpha})}{\partial m_{n,k}} + \sum_{k=1}^{2N_G(N_R-1)} \delta \alpha_k \frac{\partial f_{ij}(\mathbf{m}_n, \boldsymbol{\alpha})}{\partial \alpha_k}, \quad (37)$$

which also involves the known expressions for $\delta m_{n,k}$ [20]. As in the quark case, this alternative OS renormalization prescription is physical and satisfies all three criteria. At the same time, it guarantees that Eq. (31) is fulfilled. However, similarly to the quark case, it breaks the symmetry between the lepton families. For this reason, we advocate the OS renormalization prescription presented in the first part of this section.

4 Conclusions

We proposed physical OS renormalization conditions for the CKM matrix of the SM and for the lepton mixing matrices of a favourable class of see-saw-type Majorana-neutrino theories, in which all fermion generations have the same structure. Apart from being UV finite, the resulting renormalized mixing matrices are gauge independent and (pseudo)unitary.

In the SM, our strategy was to select a suitable charged-current process, such as the decay $W^+ \rightarrow u_i \bar{d}_j$, and to match the loop-corrected T -matrix element of the full theory with the one of a reference theory, in which the u_i and d_j quarks are arranged to belong

to the same fermion generation and quark mixing is switched off. Since the latter is UV finite and gauge independent, this equality defines a counterterm for the CKM matrix that has the correct UV divergences and is gauge independent. However, the corresponding renormalized CKM matrix is not unitary. In a second step, unitarity is installed by a UV-finite, gauge-independent shift of the counterterm.

This procedure was then generalized to see-saw-type Majorana-neutrino theories. Here, one faces the complication that the lepton mixing matrices do not only depend on mixing angles and phases, which are responsible for interfamily mixing, but also on neutrino masses. Thus, nullifying the mixing angles and phases leads us to a reference theory with intrafamily mixing, rather than zero mixing. The lepton mixing matrices of this reference theory can be renormalized by shifting the bare neutrino masses according to the usual OS renormalization scheme. In this way, the texture zero in the neutrino mass matrix is preserved, and so is gauge invariance. The matching and unitarization procedures can then be carried out in analogy to the SM case. In general Majorana-neutrino theories without see-saw mechanism, where the texture zero is traded against an additional Higgs triplet, the lepton mixing matrices can be taken to be independent of neutrino masses, which considerably simplifies the renormalization procedure.

We explicitly worked at one loop, but we believe that our formalism is likely to carry over to higher orders. From the phenomenological point of view, a two-loop analysis of the CKM matrix does not appear to be necessitated by the experimental precision to be achieved in the foreseeable future. In the Majorana-neutrino case, such a two-loop analysis lacks motivation before the neutrino puzzle is solved and the underlying pattern of the neutrino sector is fully understood.

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